Mathematics Methods
Unit 3

Binomial distribution

1.	**Bernoulli trial**
	Bernuoli trial or binomial trial propeties (charactesristics): • Each of the trials has only two outcomes: success,p or failure,q • Trials are independent of each other (outcomes of previous trial has no influence on the outcome of the following trial • Trials are discrete random variable Bernuoli trial can be represented by: $$p + q = 1$$ p: probability of success q: probability of failure
2.	**Bernoulli random variable**
	(a) Mean/ expected value
	Formula: $$\mu = p$$ Derivation of formula: $E(X) = \sum x \times P(X = x)$ $= P(X = 0) + P(X = 1)$ $= p(1) + q(0)$ $= p$
	Example 1: A bag contains two types of cards: black card and gold card. There are one gold card and three black cards. The random variable x is defined as the number of black card(s) drawn. There is only a chance in which the cards can be drawn. Determine the mean of the distribution. Example 2: Given that $Z = 1$ when a jocker card is drawn while $Z = 0$ for all other cards drawn. Calculate the expected value of Z in a traditional deck of cards.

(b) Variance

Formula:

$$\sigma^2 = pq$$

Derivation of formula:

$$\begin{aligned} Var(X) &= E(X^2) - [E(X)]^2 \\ &= p - p^2 \\ &= p(1-p) \\ &= pq \end{aligned}$$

$$\begin{aligned} E(X^2) &= \sum x^2 \times P(X = x) \\ &= x^2\, P(X = 1) + x^2\, P(X = 0) \\ &= 1^2 P(X = 0) + 0^2 P(X = 1) \\ &= p \end{aligned}$$

$$[E(X)]^2 = p^2$$

Example 1:
Calculate the variance of a distribution if the probability of success is 0.2 while the probability of failure is 0.8.

Example 2:
Based on the previous statistics, a school has 4% of students scoring straight A's in the year 11 examination. The school principal made a forecast that the forecast students obtaining straight A's is the same as the previous year.
$Y = 1$ is defined when a student falls under the straight A's category while $Y = 0$ is defined when a student falls under other categories other than straight A's. Find the variance of the distribution.

(c) Standard deviation

Formula:

$$\sigma = \sqrt{pq}$$

Derivation of formula:

$$\begin{aligned} Std(X) &= \sqrt{Var(X)} \\ &= \sqrt{pq} \end{aligned}$$

Example:
The probability distribution of an event is shown in the table below:

x	0	1
$P(X = x)$	0.4	0.6

Find the standard deviation of the distribution.

3. Binomial distribution

Definition: Binomial distribution is a type of discrete random variable (specific), counting the probability of an event over a fixed number of trials.

How binomial distribution is formed?

$$\text{Bernoulli distribution} \xrightarrow{\text{Repeated } n \text{ number of times}} \text{Binomial distribution}$$

Binomial distribution can be denoted by:

$$X{\sim}B(n, p)$$

n: number of trials/ repetition
p: probability of success

Binomial distribution properties (characteristics):

- Has only two outcomes (success,p or failure,q)
- Probability is constant for success and failure
- It is repeated for a number of times/ trials
- Trials are independent of one and another

(a) Probability

Parameters

$$X{\sim}B(n, p)$$

n: number of repitition
p: probability of success

Formula

$$P(X = r) = {}^nC_r p^r q^{n-r}$$

Where:
X: discrete random variable in a binomial distribution
n: number of trial
r: number of success
$n - r$: number of failure
p: probability of success
q: probability of failure

Example 1:
Given that $X \sim B(8, 0.5)$, find:

$P(X > 2)$	
$P(X \leq 2)$	
$P(X \leq 5 \mid X \geq 1)$	

Example 2:
In a survey, 20% of the students likes mathematics. If a random sample of 4 students are choosen, find the probabilty that two of them like mathematics.

Example 3:
Azri is salesman. It is known that the probability of getting a potential customer in sales is 0.05. What is the least number of calls that must be made to ensure that the probability of making at least 2 sales is more than 90%?

Example 4:
A random variable is binomially distributed. It's variance is 2 while mean is 6. Find $P(X = 3)$.

(b) Mean

Formula:

$$\mu = np$$

Derivation of formula:

$$E(X) = \sum x \times P(X = x)$$

$$= \sum_{x=0}^{n} x \times P(X = x)$$

$$= \sum_{x=0}^{n} x \times {}^{n}C_{r} p^{r} q^{n-r}$$

$$= \sum_{x=0}^{n} x \times \frac{n!}{x!\,(n-x)!} \times p^{x}(1-p)^{n-x}$$

$$= \sum_{x=1}^{n} \frac{n!}{(x-1)!\,(n-x)!} \times p^{x}(1-p)^{n-x}$$

$Let\ R = x - 1\ \&\ m = n - 1,$

$$= \sum_{R=0}^{m} \frac{(m+1)!}{R!\,(m-R)!} \times p^{R+1}(1-p)^{m-R}$$

$$= (m+1)p \sum_{R=0}^{m} \frac{m!}{R!\,(m-R)!} \times p^{R}(1-p)^{m-R}$$

$$= np \ * \sum_{R=0}^{m} \frac{m!}{R!\,(m-R)!} \times p^{R}(1-p)^{m-R}$$

$$= np(1)$$

$$= np$$

*Binomial theorem,

$$(a+b)^{m} = \sum_{R=0}^{m} \frac{m!}{R!\,(m-R)!} \times a^{R}(b)^{m-R}$$

$Let\ a = p, \qquad b = 1 - p$

$$\sum_{R=0}^{m} \frac{m!}{R!\,(m-R)!} \times p^{R}(1-p)^{m-R} = \sum_{R=0}^{m} \frac{m!}{R!\,(m-R)!} \times a^{R}(b)^{m-R}$$

$$= (a+b)^{m}$$

$$= (p + 1 - p)^{m}$$

$$= 1^{m}$$

$$= 1$$

Example 1:

Kobauro Ltd. predicts that out of the monthly production batch of motherboards, 2% are defective due to various reasons. Find the expected number of defective motherboards in a sample of 400 motherboards.

Example 2:
X is a discrete random variable such that $X \sim B(n, p)$. If the value of $q = 0.1$ and the variance is 34, find the mean of distribution.

(c) Variance

Formula:

$$\sigma^2 = npq$$

Derivation of formula:

$$\begin{aligned}
\sigma^2 &= E(X^2) - [E(X)]^2 \\
&= E(X^2) - E(X) + E(X) - [E(X)]^2 \\
&= E[x(x-1)] + E(X) - [E(X)]^2 \\
&=* \, n(n-1)p^2 + np - n^2p^2 \\
&= n^2p^2 - np^2 + np - n^2p^2 \\
&= -np^2 + np \\
&= np(1-p)
\end{aligned}$$

$Since\ q = 1 - p,$

$$= npq$$

$$* \ E[x(x-1)] = \sum_{x=2}^{n} \frac{n!}{(x-2)!\,(n-x)!} \times p^x(1-p)^{n-x}$$

$Let\ R = x - 2\ \&\ m = n - 2,$

$$\begin{aligned}
&= n(n-1)\sum_{R=0}^{m} \frac{m!}{R!\,(m-R)!} \times p^{R+2}(1-p)^{m-R} \\
&= n(n-1)p^2\sum_{R=0}^{m} \frac{m!}{R!\,(m-R)!} \times p^{R}(1-p)^{m-R} \\
&= n(n-1)p^2(p+1-p)^m \\
&= n(n-1)p^2
\end{aligned}$$

Example 1:
Michael who sells “economy rice” predicts that out of the daily dish cooked, 26% are left unsold. Find the variance of the leftover dishes at the end of the day if Michael cooks 25 dishes that day.

Example 2:
Kaishan throws a biased die 20 times and the number of 2’s seen is 8 times. Find the variance for the appearance of the number, 2.

(d) Standard deviation

Formula:

$$\sigma = \sqrt{npq}$$

Derivation of formula:
(same as of variance)

$$\begin{aligned}\sigma^2 &= npq \\ &= \sqrt{npq}\end{aligned}$$

Example 1:
Given that $X \sim B(n, p)$ and the value of $q = 0.9$ while $n = 5$. Calculate the standard deviation.

Example 2:
Suppose that a fair coin is flipped 30 times. Calculate the standard deviation.

END